IWR – Universität Heidelberg
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Due date: **13.12.2013**

Homework No. 7
Numerical Methods for PDE, Winter 2013/14

Problem 7.1: H^1- and L^2-Error Estimates (6 points)

Consider the Neumann problem

$$\begin{aligned} -\Delta u + u &= f, && \text{in } \Omega, \\ \partial_n u &= 0, && \text{on } \partial\Omega, \end{aligned}$$

where Ω is a convex polygonal domain in $\mathbb{R}^2$.

We want to use a conforming FE method with piecewise linear finite elements ($V_h \subset H^1(\Omega)$) to approximate the above problem.

(a) Formulate the variational equation and its approximation, and name the ansatz spaces V_h.

(b) Derive an error estimate in the H^1-norm.

(c) Derive an error estimate in the L^2-norm by Aubin's and Nitsche's duality argument.

Problem 7.2: Special A Posteriori Error Estimators (6 points)

Consider the general Neumann problem

$$\begin{aligned} -\nabla \cdot (\alpha(x)\nabla u(x)) + \gamma(x)u(x) &= f(x) && \text{in } \Omega \\ n(x) \cdot (\alpha(x)\nabla u(x)) &= g(x) && \text{on } \partial\Omega \end{aligned}$$

with smooth data functions α, γ, f, g and $\alpha(x) \geq \tilde{\alpha} > 0$ and $\gamma(x) \geq \tilde{\gamma} > 0$. The domain $\Omega \subset \mathbb{R}^2$ is again a convex polygon.

(a) Derive the weak formulation as usual and discretize the equation with piecewise linear finite elements.

(b) Formulate the dual problem for a general error functional $J(\varphi)$.

(c) Derive an error representation for the error $e_h = u - u_h$ in the functional $J(\cdot)$ by cellwise dual weighted residuals analogously to the proceeding in the lecture.

(d) Derive an a posteriori error estimation for the error in the L^2-norm $\|e_h\|_{L^2(\Omega)}$.

Hint: Use the nodal interpolant $\psi_h := \Pi_{\mathcal{Z}} z$ and their properties.