IWR – Universität Heidelberg
Prof. Dr. Guido Kanschat

Due date: **(24.1.2014)**

Homework No. 11
Numerical Methods for PDE, Winter 2013/14

Note: In this last homework, we use the techniques we have developed for the poisson equation and apply them to a now problem, the plate equation. The points for this homework are all bonus points, therefore it is not mandatory to hand the results in. Nevertheless, it summarizes most of the techniques acquired in this class. Therefore, we highly recommend preparing for the discussion of the problems in the last two tutorials.

Problem 11.1: Plate Equation

Consider the non homogeneous biharmonic equation

$$\begin{aligned} \Delta^2 u \equiv \tfrac{\partial^4}{\partial x^4} u + 2 \tfrac{\partial^4}{\partial x^2 \partial y^2} u + \tfrac{\partial^4}{\partial y^4} u &= f && \text{in } \Omega, \\ u &= 0 && \text{on } \partial\Omega, \\ \partial_n u &= 0 && \text{on } \partial\Omega. \end{aligned}$$

(a) Formulate the problem weakly. Do this in a way, that the resulting bilinear form is symmetric. Choose an appropriate function space V.

(b) Prove the existence of a unique weak solution in V.

Problem 11.2: The Bogner-Fox-Schmitt element

The Bogner-Fox-Schmitt element (see Figure 1) is defined on the reference square $\hat{K} = [0,1]^2$ as the space of bicubic shape functions Q_3 with the following set of node functionals associated to each vertex p_i of the square (see also the figure):

(a) The function value $u(p_i)$.

(b) The two partial derivatives $\partial_\xi u(p_i)$ and $\partial_\eta u(p_i)$.

(c) The mixed second derivative $\partial_\xi \partial_\eta u(p_i)$.

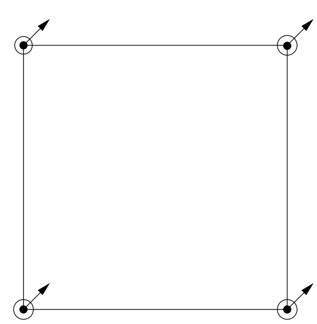

Figure 1: The Bogner-Fox-Schmitt element. Point values: $\bullet$, first derivatives: $\bigcirc$, mixed second order derivatives: $\nearrow$

(a) Show that the element is unisolvent (the whole proof might be too much work, but make sure you point out the main arguments)

(b) Show that the element yields a continuously differentiable function space on a uniform, Cartesian mesh. Argue, that it is thus conforming with the space $H^2(\Omega)$,

(c) Use the last homework to show that the (spectral) condition number of the stiffness matrix has the property $\mathcal{O}(h^{-4})$ (pointing out the main arguments is sufficient).