Westfälische Wilhelms-Universität Münster
Applied Mathematics Münster

Interactive Simulations Using Parallel, Event-Driven Localized Reduced Basis

PDESoft 2014 - Heidelberg

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July 15, 2014

Outline

1. Time Harmonic Maxwell's Equations
2. Software Design
3. Model Order Reduction
4. Numerical Example
5. Outlook

Simulating a Chip Carrier in a Flip Chip Package

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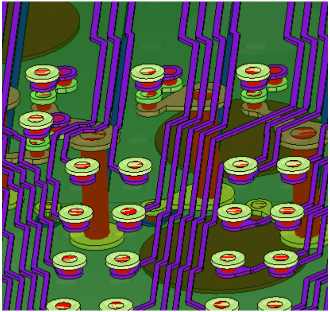
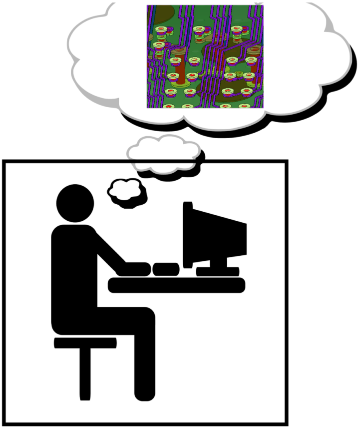
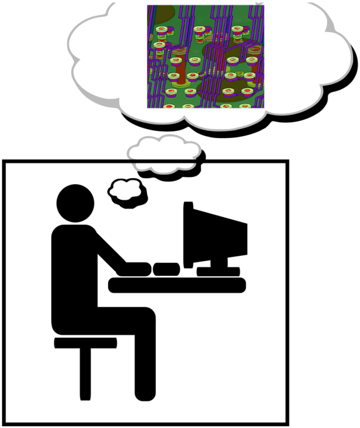


Non-Parametric Geometry Changes



Envision engineer working on design

Non-Parametric Geometry Changes



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Multi-query setting

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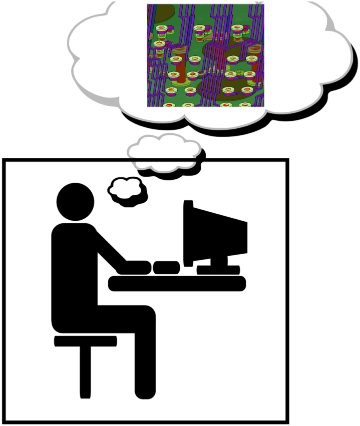

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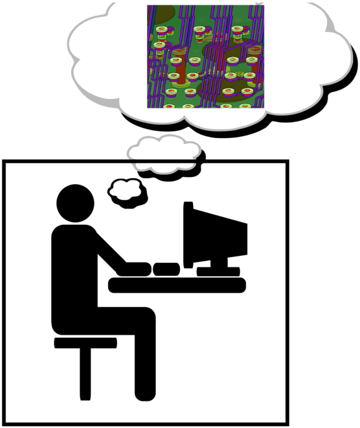
Multi-query setting

Properties of changes:

1. very localized

Non-Parametric Geometry Changes



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Properties of changes:

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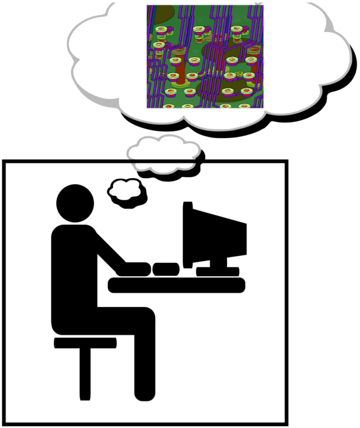

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Multi-query setting

Properties of changes:

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2. unforeseen

Cluster often available.
Cloud always available.

Time Harmonic Maxwell's Equations

$$\nabla \times \frac{1}{\mu} \nabla \times E - \omega^2 \epsilon E = -i\omega j \qquad \text{in} \quad \Omega \tag{1}$$

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$$\omega \in [0, 10^{10}]$$

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- Simulation in a frequency range, e.g.

$$\omega \in [0, 10^{10}]$$

- Dirichlet boundary:

$$E \times n = g \quad \text{on } \partial\Omega \quad (= 0 \text{ on most of } \partial\Omega)$$

Part I:

Software Design for Interactive Simulations

Software Design for Interactive Applications

Design of interactive applications is well understood:

- Event driven
- Signal/Slot based (like e.g. Qt / Boost.Signals)

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Agenda

1. Signal/Slot in cluster
2. Implement FEM-Solver in terms of it

Signal/Slot in Cluster

HPX by Ste||ar Group at Louisiana State University[1]

HPX is
“A general purpose C++ runtime system for parallel and distributed applications of any scale”

[1]The STE||AR Group’ (stellar.cct.lsu.edu, github.com/STEllAR-GROUP/hpx)

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What we need:

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→ We reimplemented what we need in Python.

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Publishers and Subscribers

Objects can have publishers. Other objects can subscribe.
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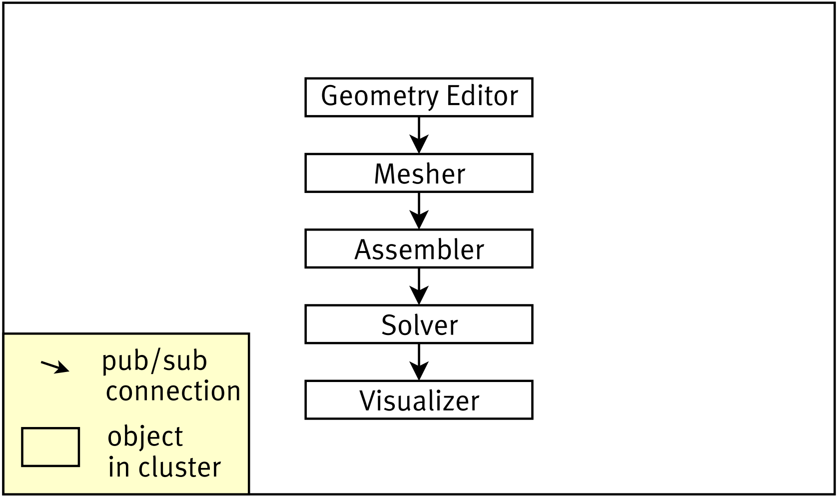
Subscriber interface:

- Subscriber(publisher_id, validation_callback, invalidation_callback)
- get_data()
- is_valid()

Event-Driven Finite Element Solver (simple)



Add Domain Decomposition

Without Domain Decomposition:

- Little parallelism
- Large amounts of data transferred

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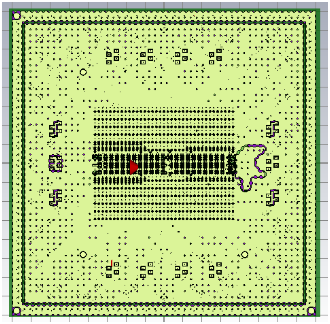

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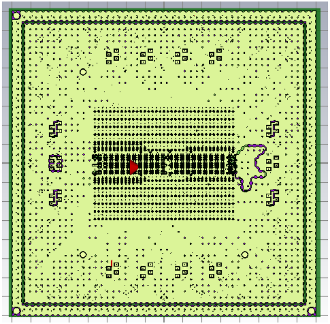
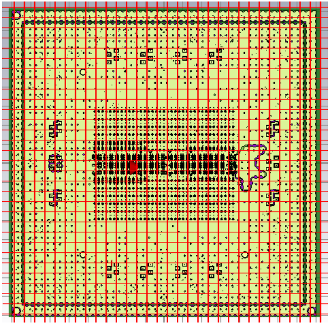
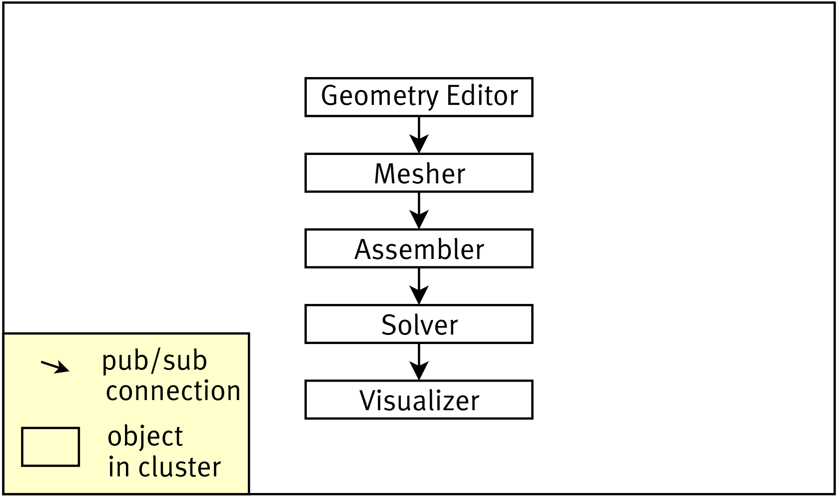




Event Driven Finite Element Solver



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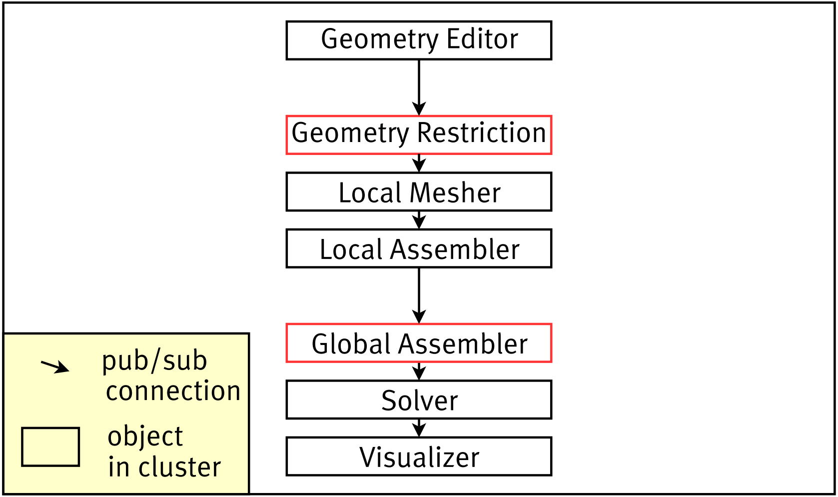

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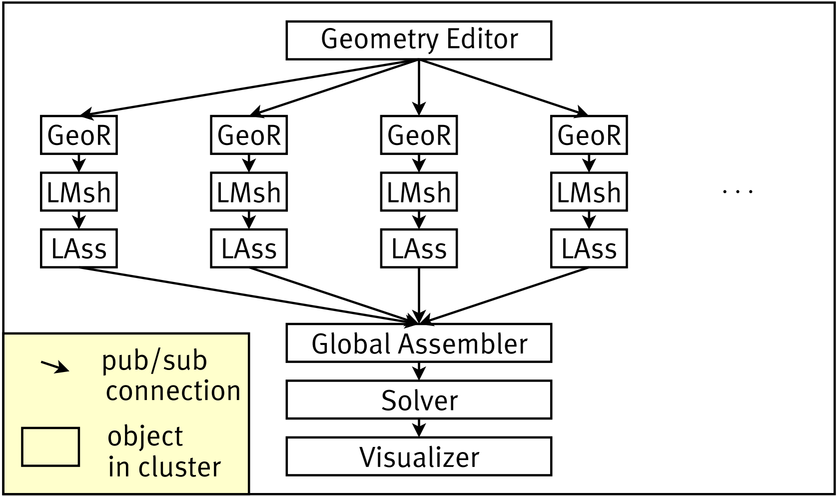

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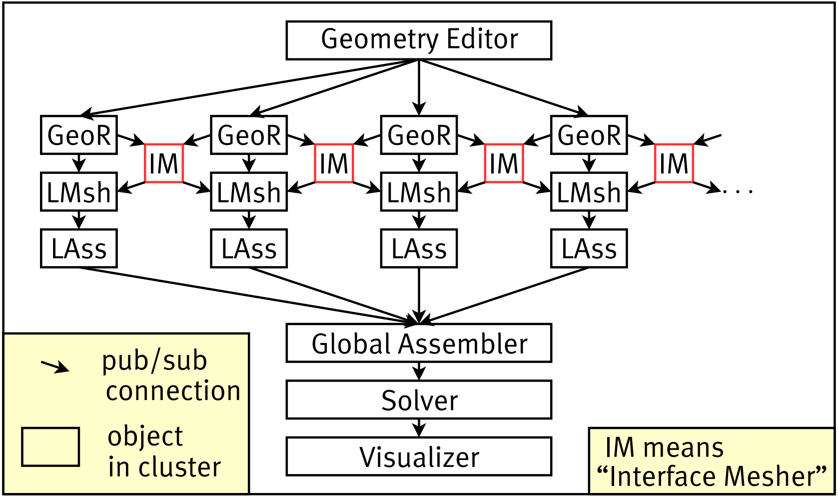

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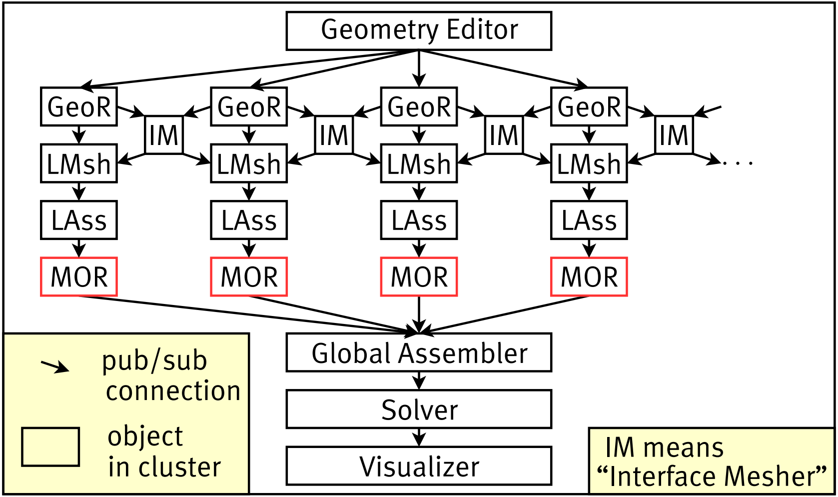

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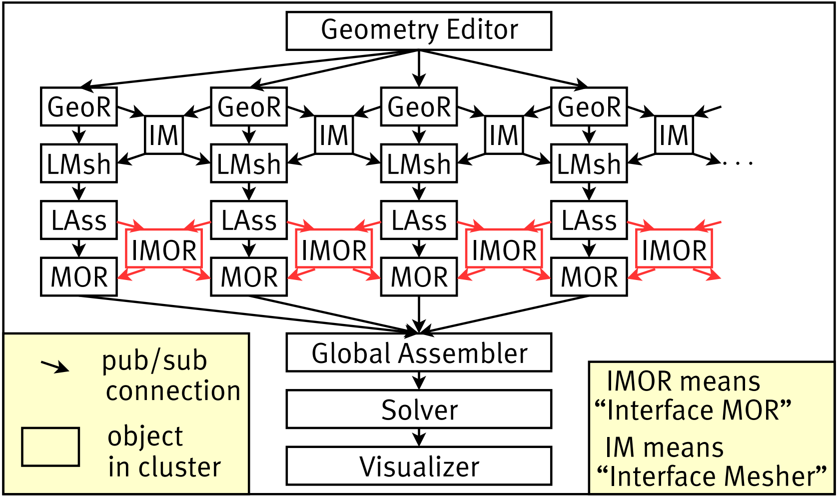

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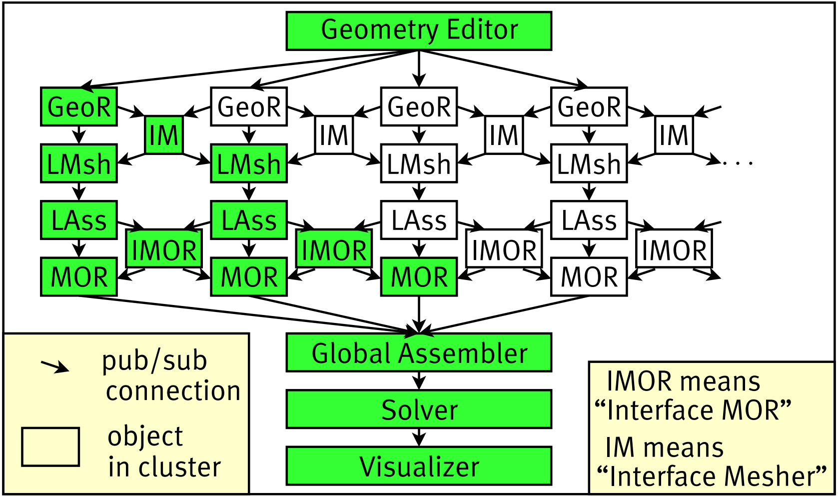
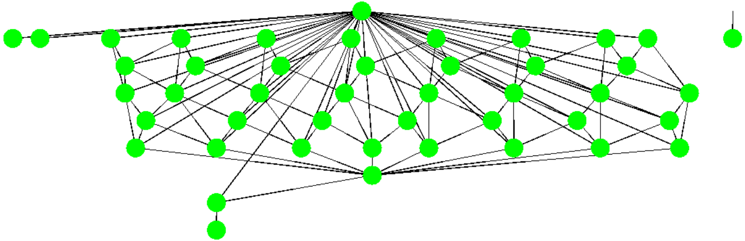


Real World Dependency Graphs

Dependency graph for 8 domains:



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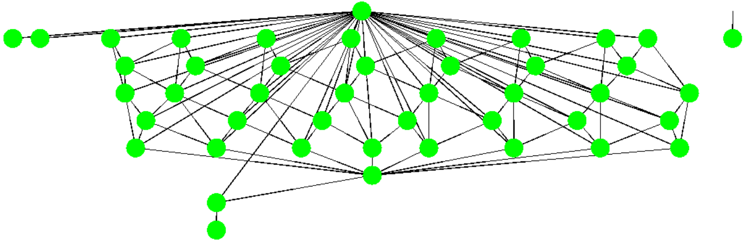

Dependency graph for 64 domains:

Dependency graph for more domains not shown

Part II:
Generating Subdomain Spaces

Standard Reduced Basis

Problem

Find u_ω in V such that

$$a_\omega(u_\omega, v) = f_\omega(v) \qquad \forall\, v \in V$$

Reduced Basis Approach

Construct subspace $\widetilde{V} \subset V$ with $\dim(\widetilde{V}) \ll \dim(V)$,
find $\widetilde{u}_\omega$ in $\widetilde{V}$:

$$a_\omega(\widetilde{u}_\omega, \widetilde{v}) = f_\omega(\widetilde{v}) \qquad \forall\, \widetilde{v} \in \widetilde{V}$$

Two Main Questions

The two main questions for this approach are:

- How to construct the reduced space $\widetilde{V}$?
- How to control the error $||u_\omega - \widetilde{u}_\omega||_V$?

For standard (not localized) RB, rich theory exists.

Localization by Grouping of Ansatzfunctions

Decomposition of Ansatz Space

The space V is the direct sum of subspaces:

$$V = \Big(\bigoplus_i V_{D_i}\Big) \oplus \Big(\bigoplus_j V_{I_j}\Big) \qquad V_{D_i} \subset V, \qquad V_{I_j} \subset V \tag{2}$$

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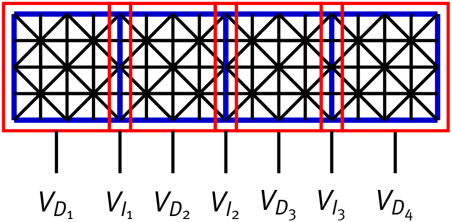
Four Domain Example:



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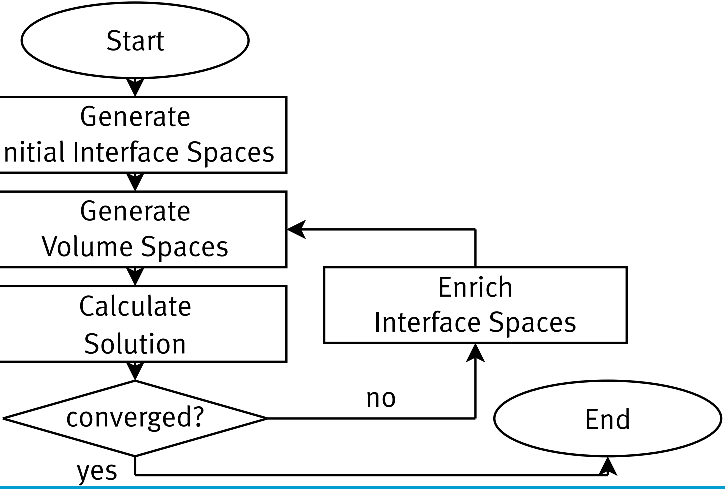
Localized Reduced Basis

For each V_{D_i} and V_{I_j} construct subspaces $\widetilde{V}_{D_i} \subset V_{D_i}$ and $\widetilde{V}_{I_j} \subset V_{I_j}$ with $\dim(\widetilde{V}_{D_i}) \ll \dim(V_{D_i})$ and $\dim(\widetilde{V}_{I_j}) \ll \dim(V_{I_j})$, find $\widetilde{u}_\omega$ in $\widetilde{V}_{LRB} := \big(\bigoplus_i \widetilde{V}_{D_i}\big) \oplus \big(\bigoplus_j \widetilde{V}_{I_j}\big) \subset V$:

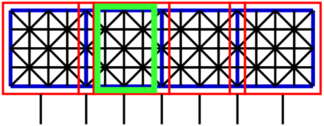
$$a_\omega(\widetilde{u}_\omega, \widetilde{v}) = f_\omega(\widetilde{v}) \qquad \forall\, \widetilde{v} \in \widetilde{V}_{LRB}$$

Enrichment Algorithm



Construction of Volume Subspaces

- easy, if interface subspaces are available
- construct space containing solutions for all interface conditions



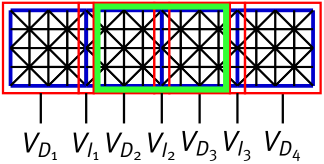
Construction of volume subspaces

E.g. for V_{D_2}:

$$\widetilde{V}_{D_2} := \operatorname{span}(\{\psi \in V_{D_2},\ a_\omega(\psi + \varphi_1 + \varphi_2, v) = f_\omega(v) \quad \forall\, v \in V_{D_2},\ \varphi_1 \in \widetilde{V}_{I_1}, \varphi_2 \in \widetilde{V}_{I_2}\})$$

Construction of Interface Subspaces

- research topic
- right now: interface basis enriched by solving patch problems
- steered by localized error indicator



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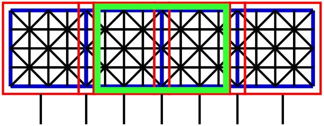

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Projection Operator

P_{D_i}, P_{I_j} is projection to V_{D_i}, V_{I_j}:

$$P_{D_i} : V \to V_{D_i} \qquad P_{I_j} : V \to V_{I_j}$$

defined by

$$v = \sum_i P_{D_i}(v) + \sum_j P_{I_j}(v) \quad \forall v \in V$$

Construction of Interface Subspaces

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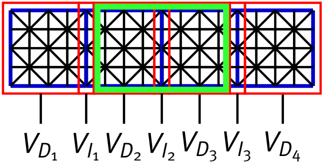


Construction of interface subspaces

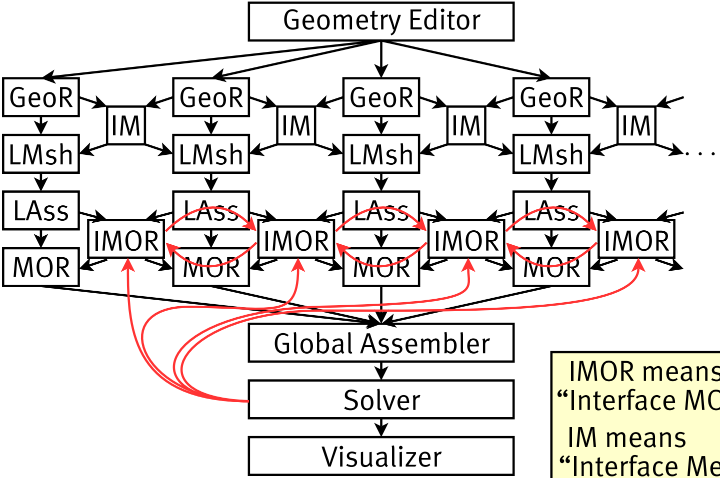
E.g. for V_{I_2}, solve in $V_{\text{patch},I_2} := V_{D_2} \oplus V_{I_2} \oplus V_{D_3}$

$$\widetilde{V}_{I_2,i+1} := \widetilde{V}_{I_2,i} \oplus \text{span}(P_{I_2}(\psi))$$

where ψ in V_{patch,I_2} is the solution of

$$a_\omega(\psi + P_{I_1}(\widetilde{u}_{\omega,i}) + P_{I_3}(\widetilde{u}_{\omega,i}), v) = f_\omega(v) \quad \forall\, v \in V_{\text{patch},I_2}$$

Additional Data Dependencies

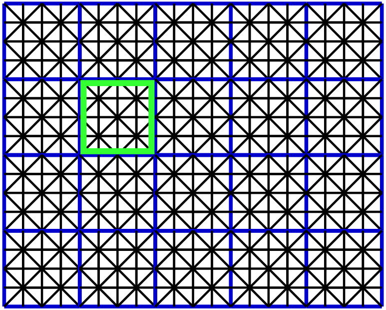


IMOR means “Interface MOR”

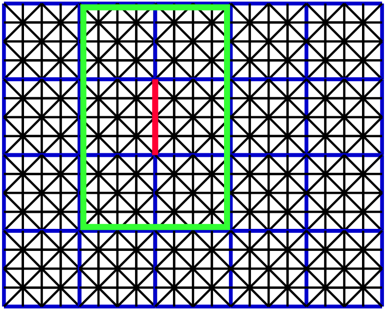
IM means “Interface Mesher”

Patches in 2D

for volume subspace



for interface subspace



Similar approaches in Literature

LRBMS, GMsFEM, PR-SCRBE:

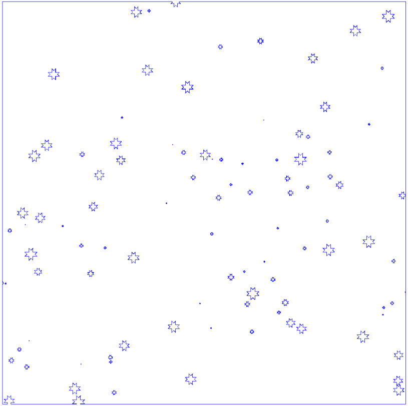
F. Albrecht, B. Haasdonk, M. Ohlberger, and S. Kaulmann.
The localized reduced basis multiscale method.
Proceedings of Algoritmy 2012, Conference on Scientific Computing, Vysoke Tatry, Podbanske, September 9-14, 2012, pages 393–403, 2012.

Yalchin Efendiev, Juan Galvis, and Thomas Y. Hou.
Generalized multiscale finite element methods (gmsfem).
January 2013.

Jens L Eftang and Anthony T Patera.
A port-reduced static condensation reduced basis element method for large component-synthesized structures: approximation and a posteriori error estimation.
Advanced Modeling and Simulation in Engineering Sciences, 1(1):3, 2014.

Numerical Example

test geometry: 2D metal box with random metal stars



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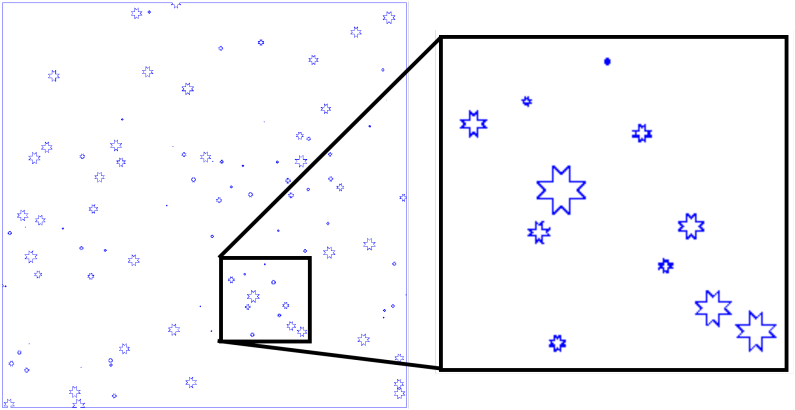

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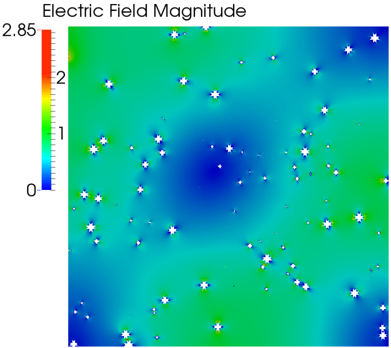
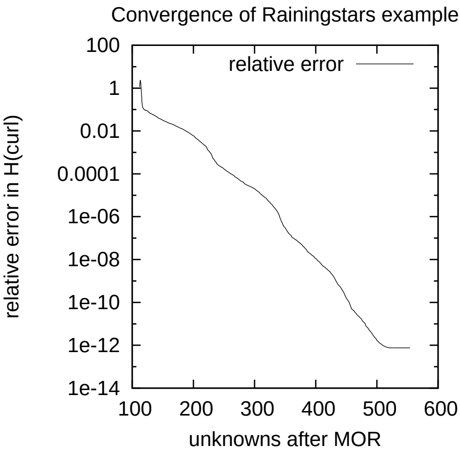


Convergence with Iterative Enrichment



- 183 219 unknowns in full system
- 8x8 domain decomposition
- 112 internal interfaces
- 1e-5 at 313 dofs
- 1e-10 at 455 dofs

Outlook

We are planning to:

- implement rigorous a-posteriori error estimates[2]
- go for 3D
- evaluate more strategies for interface reduction

[2] K Smetana, A new certification framework for the port reduced static condensation reduced basis element method. Computer Methods in Applied Mechanics and Engineering

Acknowledgements

Many thanks to...

CST - Computer Simulation Technology AG[3]

... for sponsoring my research.

[3] www.cst.com