Adaptive solution strategies for coupled porous media flow / free flow systems

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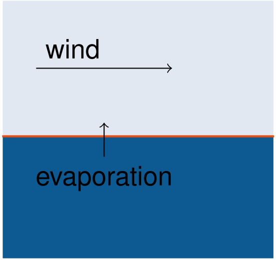
Coupling free flow / porous-medium flow

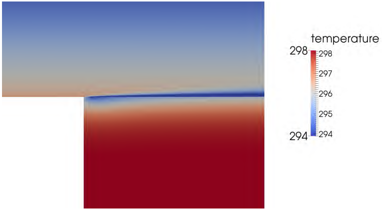
General idea

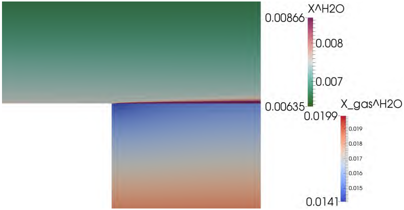


free flow, Navier-Stokes
1 phase, 2 components, temperature

sharp interface

porous-medium / Darcy flow
2 phases, 2 component, temperature





Coupling free flow / porous-medium flow

Applications

Potash evaporation ponds

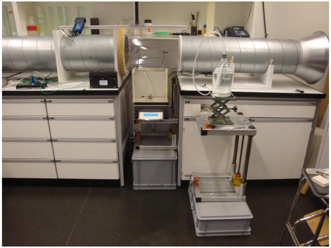
Freeze-drying


Agricultural Research Service, k4500-12

Salinization in California

Stokes / Darcy coupling

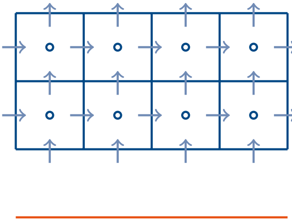
- Coupling of Stokes and Darcy *[Mosthaf et al. 2011, Baber et al. 2012]*
- 2 phases, 2 components, temp. / 1 phase, 2 components, temp.
- box scheme, implicit Euler
- assemble in single matrix, Newton, SuperLU
- oscillations with Navier-Stokes

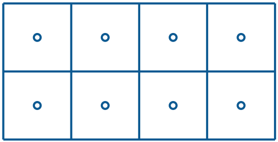


In collaboration with Dani Or, ETH Zurich

Navier-Stokes / Darcy coupling

Schemes





Navier-Stokes / Darcy coupling

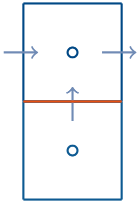
Coupling

Continuity of normal stresses

$$p^{\mathsf{pm}} = p^{\mathsf{ff}} + n_\Gamma \left(\varrho v v^{\mathsf{T}} - \varrho \nu \nabla v\right)^{\mathsf{ff}} n_\Gamma$$

Continuity of normal mass fluxes

$$v^{\mathsf{pm}} n_\Gamma = v^{\mathsf{ff}} n_\Gamma$$



Beaver-Joseph-Saffman condition

$$0 = \left(\alpha_{\mathsf{BJ}} v + \frac{\sqrt{t_{\Gamma,i}^{\mathsf{T}} K t_{\Gamma,i}}}{\nu \varrho} \tau n_\Gamma\right)^{\mathsf{ff}} \cdot t_{\Gamma,i}$$

$$= \left(\alpha_{\mathsf{BJ}} v + \sqrt{t_{\Gamma,i}^{\mathsf{T}} K t_{\Gamma,i}} \nabla v n_\Gamma\right)^{\mathsf{ff}} \cdot t_{\Gamma,i}$$

Navier-Stokes / Darcy coupling

Models and Software

Dune

DuMu^x

Navier-Stokes, 2c, ni

PDELab: 1 phase (2 components, temp., turbulence models)

Coupling

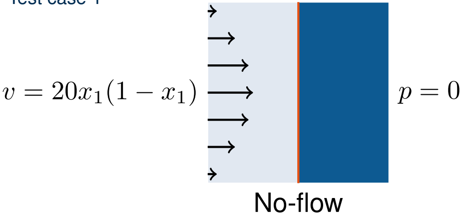
dune-multidomaingrid, -multidomain

Darcy, 2p, 2c, ni

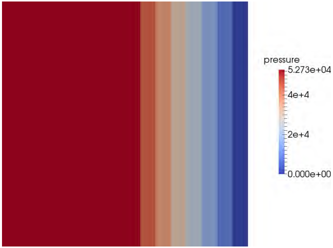
PDELab: 1 phase
(DuMu^x: 2 phases, 2 component, temp.)

Navier-Stokes / Darcy coupling

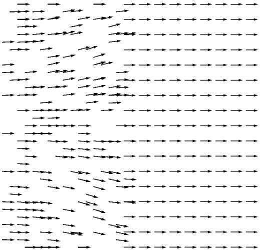
Test case 1



Test case from *[Kanschat, Rivière 2010]*



Pressure



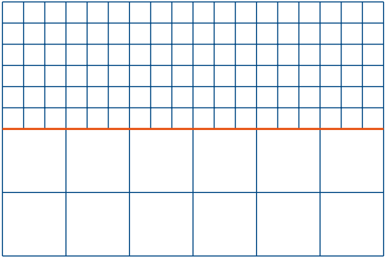
Velocity

Current status

- Compare with analytic solution *[Chidyagwai, Rivière 2011]*
- Physical laws for 2 components and temperature in Navier-Stokes
- Coupling for components and temperature
- SuperLU limits problem size
- Algebraic and k-ε turbulence models

Possible speed-up by adaptivity

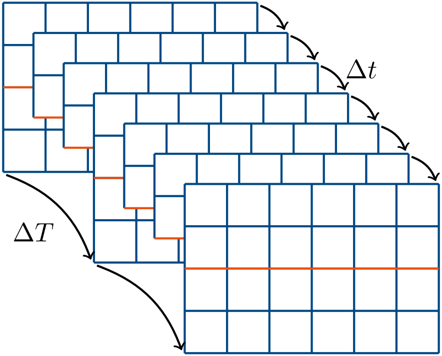
Different mesh sizes



- Coarser mesh for Darcy sufficient
- Couple with a mortar method
- Exists for Stokes / Darcy code *[Baber unpublished]*

Possible speed-up by adaptivity
Multi time stepping

- Calculate free flow with smaller time steps Δt
- Calculate complete system with with ΔT
- Unclear how conserve mass / energy
- Stokes / Darcy with 1 phase, linear *[Rybak, Magiera 2014]*



Possible speed-up by adaptivity

Decouple linear systems

Decouple linear systems using Schur complement *[Discacciati, Quarteroni 2004]*

$$Ax + By = f$$
$$Cx + Dy = g$$

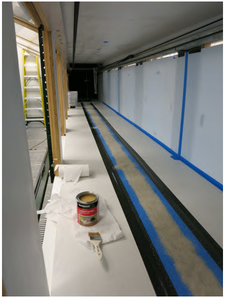
$$\left(D - CA^{-1}B\right) y = g - CA^{-1} f$$
$$Ax = f - By$$

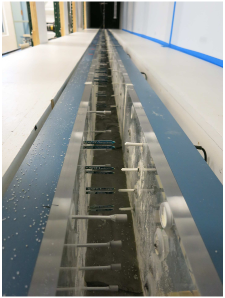
Only invert systems A and $D - CA^{-1}B$

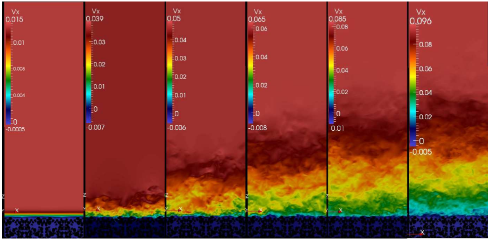
- Coupling: free flow and porous-medium flow
- Navier-Stokes: mass and momentum balance equations
- Multiple equations: mass balance equations and temperature / components

Validation and Verification

Wind tunnel experiments in collaboration with Kate Smits, Colorado School of Mines







Direct numerical simulation courtesy of Wang et. al., iRMB TU Braunschweig

Outlook

- Unstructured grids, especially non-trivial topology for coupling interface *[Ansanay-Alex et al. 2011, Verboven et al. 2006]*
- Model adaptivity
- Reconsider Beavers-Joseph condition
- Really decouple domains, iterative coupling *[Discacciati 2004]*
- Use explicit method in Navier-Stokes
- Couple with different software tools like OpenFOAM
- Scale effects up

Backup

Mass and momentum balance equations

Navier-Stokes mass balance equation

$$\frac{\partial}{\partial t}\varrho + \mathrm{div}(\varrho v) - q_p = 0$$

Navier-Stokes momentum balance equation

$$\frac{\partial}{\partial t}(\varrho v) + \mathrm{div}\left(\varrho v v^{\mathsf{T}}\right) - \mathrm{div}(\varrho \nu \nabla v) + \nabla p - \varrho g - q_v = 0$$

Darcy flow equation

$$\Phi \frac{\partial}{\partial t}(\varrho S) - \mathrm{div}\left(\frac{K}{\nu}(\nabla p - \varrho g)\right) - q_{\mathrm{pm}} = 0$$

Backup

additional free flow equations

energy balance equation

$$\frac{\partial}{\partial t}(\varrho u) + \operatorname{div}(\varrho v h - \lambda \nabla T) = q_{\mathsf{T}}$$

transport equation

$$\frac{\partial}{\partial t}(\varrho X) + \operatorname{div}(\varrho v X - D_{\mathsf{steam}} \varrho \nabla X) = q_{\mathsf{steam}}$$

Backup

additional Darcy flow equations

energy balance equation

$$\sum_{\alpha \in \{\mathsf{g,l}\}} \Phi \frac{\partial}{\partial t} \left(\varrho_\alpha u_\alpha^\kappa S_\alpha \right) + (1 - \Phi) \frac{\partial}{\partial t} \left(\varrho_{\mathsf{pm}} c_{\mathsf{pm}} T \right)$$

$$+ \operatorname{div} \left(\sum_{\alpha \in \{\mathsf{g,l}\}} \varrho_\alpha h_\alpha v_\alpha - \lambda_{\mathsf{pm}} \nabla T \right) = q_T$$

transport equation

$$\sum_{\alpha \in \{\mathsf{g,l}\}} \Phi \frac{\partial}{\partial t} \left(\varrho_\alpha X_\alpha^\kappa S_\alpha \right) + \operatorname{div} \left(\sum_{\alpha \in \{\mathsf{g,l}\}} \left(\varrho_\alpha v_\alpha X_\alpha^\kappa - D_{\alpha,\mathsf{pm}}^\kappa \varrho_\alpha \nabla X_\alpha^\kappa \right) \right)$$

$$= \sum_{\alpha \in \{\mathsf{g,l}\}} q_\alpha^\kappa$$

Backup

coupling conditions for thermal equilibrium

temperature continuity, heat flux continuity

$$(T)_{\mathsf{ff}} = (T)_{\mathsf{pm}}$$
$$n \cdot (\varrho v h - \lambda \nabla T)_{\mathsf{ff}} = -n \cdot \left(\varrho_{\mathsf{g}} v_{\mathsf{g}} h_{\mathsf{g}} + \varrho_{\mathsf{l}} v_{\mathsf{l}} h_{\mathsf{l}} - \lambda_{\mathsf{pm}} \nabla T\right)_{\mathsf{pm}}$$

Backup

coupling conditions for chemical equilibrium

continuity of mass fractions, continuity of the component fluxes

$$(X^\kappa)_{\mathrm{ff}} = \left(X^\kappa_{\mathrm{g}}\right)_{\mathrm{pm}} \tag{1}$$

$$n \cdot (\varrho v X^\kappa - D \varrho \nabla X^\kappa)_{\mathrm{ff}} = -n \cdot \left(\sum_{\alpha \in \{\mathrm{g},\mathrm{l}\}} (\varrho_\alpha v_\alpha X^\kappa_\alpha - D_{\alpha,\mathrm{pm}} \varrho_\alpha \nabla X^\kappa_\alpha) \right)_{\mathrm{pm}} \tag{2}$$